Particle transport and rotation damping due to stochastic magnetic field lines

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It is suggested that sound wave propagation along stochastic magnetic field lines contributes to particle transport, described by a quasilinear diffusion coefficient $D = D_m c_s$, where D_m is the magnetic field line diffusion coefficient [Nucl. Fusion **6**, 297 (1966)]. In this calculation, the perturbed magnetic field $\delta\mathbf{B}$ is taken to be specified, either due to global modes or due to external perturbations, such as toroidal ripple, divertor coils, or an ergodic magnetic limiter. The transport, which proceeds by the propagation of electrostatic sound waves along the stochastic field lines, is *not* intrinsically ambipolar. The importance of this process is that it represents the shorting out of a radial electric field in the presence of δB_r, and therefore provides a mechanism for damping of plasma rotation, and may therefore be an important factor for L–H transition in tokamaks. This part of particle transport and momentum transport are therefore related and are of the same order of magnitude. However, experiments (without an ergodic magnetic limiter) appear to indicate that another source of particle transport, perhaps by $\mathbf{E}\times\mathbf{B}$ advection due to electrostatic modes, can dominate the particle transport due to sound wave propagation along stochastic field lines.

I. INTRODUCTION

The subject of energy confinement in a tokamak with stochastic field lines, or destroyed magnetic surfaces, has been of interest for a long period of time.[1–4] In the collisionless limit, the heat transport is given[1] by a quasilinear diffusion coefficient $D \approx D_m v_e$, where D_m is the magnetic field line diffusion coefficient[5] and v_e is the electron thermal speed. This thermal diffusion coefficient represents the free streaming of electrons along the stochastic, or chaotic, field lines. It has been recognized[1,6–9] that particle transport, on the other hand, must be influenced by ambipolar fields, since $v_i \ll v_e$.

In this paper we suggest that particle transport occurs via sound wave propagation along the stochastic field lines. Indeed, in a fully ionized plasma in a constant magnetic field, a density perturbation along the field lines is transported by sound wave propagation. (Here and throughout, we will assume that the particle transport is slower than the thermal transport for electrons and ions, so that each species can be taken to be isothermal.) This process is the analog to ambipolar diffusion in a Lorentz gas, in which momentum is exchanged with neutrals, as we shall discuss. In the present case, collisions conserve momentum, but momentum is not conserved because the waves involve ion inertia.

Because this process does not conserve local particle momenta, it is not intrinsically ambipolar. By intrinsically ambipolar we mean that the plasma maintains quasineutrality without the buildup of an ambipolar electric field. In Ref. 10 it was argued that transport due to stochastic magnetic field lines is intrinsically ambipolar if the *magnetic* perturbations are sufficiently localized that momentum is locally conserved. In the process we are describing, the ambipolar electric field associated with the transport is precisely the electrostatic field of the waves. Furthermore, since the waves are electrostatic the problem simplifies considerably: we take the magnetic field to be given, either due to global magnetohydrodynamic (MHD) modes or due to external perturbations such as toroidal ripple, divertor coils, an ergodic magnetic limiter, or other field errors.

For our model, we take an equilibrium with a three-dimensional magnetic field $\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}$, where $\mathbf{B}_0$ is axisymmetric and $\delta\mathbf{B}$ represents all the symmetry breaking perturbations. These magnetic perturbations are assumed to destroy the flux surfaces, at least in some region. We also take the current density j, temperatures T_e, T_i to be uniform. Here, z is taken to be the toroidal direction. Then a MHD resistive equilibrium is possible. That is, we have

$$\mathbf{j}\times\mathbf{B} = 0,$$

$$\mathbf{B}\cdot\nabla j_z = 0,$$

and

$$E_z = \eta j_z, \tag{1}$$

where E_z is the toroidal electric field (we use cylindrical coordinates r,θ,z with $0 < z < 2\pi R$) and the resistivity η is taken to be uniform, consistent with $T_e = \text{const}$. In this equilibrium, $E_z \hat{e}_z$ is the full electric field (there is no radial electric field E_r) and neither species rotates, $\mathbf{v}_{\perp i} = \mathbf{v}_{\perp e} = 0$. In general geometries it has been shown[11] that such three-dimensional Ohmic states with chaotic field lines must have variation of the electric and magnetic fields on a resistive time scale. However, for tokamak geometry this resistive time scale variation is even slower because of geometric effects and, in fact, does not show up in the reduced MHD equations.[12] This variation can therefore be ignored for our purposes.

II. THEORETICAL MODEL AND RESULTS

To investigate the evolution of a small density perturbation, we assume an electrostatic mode and linearize about this equilibrium to obtain

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$$m\frac{\partial\tilde{\mathbf{v}}}{\partial t}=-e\,\nabla\tilde{\phi}+e\tilde{\mathbf{v}}\times\mathbf{B}-\frac{T_i\,\nabla\tilde{n}}{n},\qquad(2a)$$

$$0=e\,\nabla\tilde{\phi}-e\tilde{\mathbf{v}}_e\times\mathbf{B}-\frac{T_e\,\nabla\tilde{n}}{n},\qquad(2b)$$

$$\frac{\partial\tilde{n}}{\partial t}=-n\nabla\cdot\tilde{\mathbf{v}}.\qquad(2c)$$

The component of the electron equation of motion, Eq. (2b), parallel to $\mathbf{B}$ gives $\mathbf{B}\cdot\nabla(e\tilde{\phi}-T_e\tilde{n}/n)=0$. By the assumption of chaotic field lines (i.e., $\mathbf{B}\cdot\nabla Q=0$ implies $Q=$ const throughout the plasma), we find that the electrons are adiabatic $e\tilde{\phi}/T_e=\tilde{n}/n$. Substituting again in Eq. (2b), we find that $\mathbf{v}_{e\perp}=0$, i.e., the electrons do not rotate. (The $\mathbf{E}\times\mathbf{B}$ and diamagnetic drifts cancel.) Also, the perpendicular component of Eq. (2a) (without the inertia term for $\omega/\omega_{ci}\ll 1$, where $\omega_{ci}=eB/m$) gives

$$\tilde{\mathbf{v}}_{i\perp}=\frac{(T_e+T_i)\mathbf{B}\times\nabla\tilde{n}}{enB^2}=\left(1+\frac{T_i}{T_e}\right)\frac{\mathbf{B}\times\nabla\tilde{\phi}}{B^2}.\qquad(3)$$

That is, the ions rotate due to $\mathbf{E}\times\mathbf{B}$ and diamagnetic drift, which are additive. The term $-n\nabla\cdot\mathbf{v}_{i\perp}\approx-n(1+T_i/T_e)\nabla(1/B^2)\cdot\mathbf{B}\times\nabla\tilde{\phi}$ on the right in Eq. (2c) is negligible because we are ignoring toroidal curvature effects. Therefore, we obtain, from the parallel component of Eq. (2a) and from Eq. (2c),

$$nm\frac{\partial\tilde{v}_\parallel}{\partial t}=-ne\hat{\mathbf{b}}\cdot\nabla\tilde{\phi}-T_i\hat{\mathbf{b}}\cdot\nabla\tilde{n}=-T\hat{\mathbf{b}}\cdot\nabla\tilde{n},\qquad(4a)$$

$$\frac{\partial\tilde{n}}{\partial t}=-n\nabla\cdot\tilde{\mathbf{v}}_\parallel=-nB\hat{\mathbf{b}}\cdot\nabla\left(\frac{\tilde{v}_\parallel}{B}\right),\qquad(4b)$$

where $B=|\mathbf{B}|$, $\hat{\mathbf{b}}=\mathbf{B}/B$, and $T=T_e+T_i$. In (nonintrinsic) ambipolar diffusion in a Lorentz gas, the equation corresponding to Eq. (4a) is $0=-ne\hat{\mathbf{b}}\cdot\nabla\tilde{\phi}-T_i\hat{\mathbf{b}}\cdot\nabla\tilde{n}-n\nu_{in}m_i\tilde{v}_\parallel$. The momentum transferred to the neutrals by the ion–neutral drag exactly corresponds to the acceleration of the ions in Eq. (4a). Clearly, the density gradient couples to ion flow mediated by the electrostatic potential in both cases, making the process nonintrinsically ambipolar.

The polarization drift $\mathbf{v}_\perp^p=-(m/eB^2)(\partial/\partial t)\nabla_\perp\phi$ leads to finite Larmor radius (FLR) and electromagnetic effects, which are negligible for the following reasons: the term $n\nabla\cdot\mathbf{v}_\perp^p$ in Eq. (4b) gives, with $e\tilde{\phi}/T_e=\tilde{n}/n$, a factor $(1+k_\perp^2\rho_s^2)$ on the left in (4b), where $\rho_s^2=(T_e/m_i)/\omega_{ci}^2$. We will assume that $k_\perp$ is small enough for such FLR effects to be negligible. The quasineutrality condition (or vorticity equation) $\nabla\cdot\tilde{\mathbf{v}}_\perp^p=-\nabla\cdot(\tilde{\mathbf{j}}_\parallel/ne)=-(1/ne)\mathbf{B}\cdot\nabla(\tilde{j}_\parallel/B)$, gives

$$\frac{nm}{B}\frac{\partial}{\partial t}\nabla_\perp^2\tilde{\phi}=\mathbf{B}\cdot\nabla\tilde{j}_z.$$

This implies that there is coupling to the Alfvén wave. However, the magnetic perturbation $\tilde{\mathbf{B}}$ induced by $\tilde{j}_z$ does not enter on the right in Eq. (4a) because there is no equilibrium density gradient. Also, the term $e\,\partial A_\parallel/\partial t$ in the component of Eq. (2b) parallel to $\mathbf{B}$ is of order $\beta\sim c_s^2/v_A^2$ relative to $e\,\nabla_\parallel\phi$.

Again, because we are ignoring toroidal curvature and finite aspect ratio effects, we can take B to be constant and find

$$\frac{\partial\tilde{v}_\parallel}{\partial t}=-\frac{c_s^2}{n}\hat{\mathbf{b}}\cdot\nabla\tilde{n},$$

$$\frac{\partial\tilde{n}}{\partial t}=-n\hat{\mathbf{b}}\cdot\nabla\tilde{v}_\parallel.\qquad(5)$$

Then, writing $u=\tilde{v}_\parallel/c_s+\tilde{n}/n$, $v=\tilde{v}_\parallel/c_s-\tilde{n}/n$, we obtain

$$\frac{\partial u}{\partial t}+c_s\hat{\mathbf{b}}\cdot\nabla u=0,$$

$$\frac{\partial v}{\partial t}-c_s\hat{\mathbf{b}}\cdot\nabla v=0.\qquad(6)$$

Thus u represents a wave propagating to the right $(\omega=k_\parallel c_s)$ and v represents a wave propagating to the left $(\omega=-k_\parallel c_s)$. Also note that the advective "flow" $\mathbf{u}_s\equiv c_s\hat{\mathbf{b}}$ is divergence-free for this equilibrium.

Applying quasilinear theory to the evolution of $\langle u\rangle$ and $\langle v\rangle$, where $\langle u\rangle=(4\pi^2R)^{-1}\int u\,d\theta\,dz$, we find

$$\frac{\partial}{\partial t}\langle u\rangle=\frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial}{\partial r}\langle u\rangle\right),$$

$$D=D_m c_s,\qquad(7)$$

where $D_m=(\delta B/B)^2 l_c$ is the magnetic field line diffusion coefficient,[5] with correlation length $l_c\sim qR$. A similar equation holds for $\langle v\rangle$. This diffusion coefficient for particle transport is of order $\sqrt{m_e/m_i}$ smaller than that for collisionless electron thermal conduction,[1] but comparable to that for ion thermal conduction, i.e., $v_i\sim c_s$ if $T_e\sim T_i$. [Semantically it would perhaps be clearer to call the process described by Eqs. (5)–(7) *density transport* rather than particle transport since it proceeds by wave propagation rather than actual transfer of particles.] Of course, the evolution of u and v, like the evolution of any passive scalar, is not fully described by quasilinear theory. The actual behavior involves the development of fine scales perpendicular to $\mathbf{B}$ as the quantities are "advected" along $\mathbf{u}_s=\pm c_s\hat{\mathbf{b}}$. However, dissipative effects, such as viscosity in Eq. (4a), cause strong decay of the high k Fourier components of u and v leading to overall decay. (We assume that dissipation is sufficient that $k_\perp^2\rho_s^2\ll 1$ holds.) The quasilinear coefficient (7) describes only the nondissipative early time behavior as the short scales are being developed and u, v are propagating along the chaotic field lines throughout the plasma.

If T_e/T_i is of order unity, sound waves in a collisionless plasma are, of course, Landau damped with $\omega_i\sim\omega_r\sim k_\parallel c_s$. If we take a representative field line that wanders over the minor radius a (or some fraction thereof), the length of this field line is $L\sim a^2/D_m$. Therefore a typical initial perturbation $\tilde{n}$ initialized constant along the magnetic surfaces corresponding to the axisymmetric field $\mathbf{B}_0$ will have

$$k_\parallel\sim\frac{1}{L}\sim\frac{D_m}{a^2}\sim\left(\frac{\delta B}{B}\right)^2\frac{qR}{a^2}.\qquad(8)$$

Landau damping, then, will be negligible if $\nu_{ii}>k_\parallel c_s$, or

$$\nu_{ii}>\left(\frac{\delta B}{B}\right)^2\frac{qRc_s}{a^2}.\qquad(9)$$

In terms of $\lambda_i\sim c_s/\nu_{ii}$, the condition is

$$\frac{qR}{\lambda_i}>\left(\frac{\delta B}{B}\right)^2\left(\frac{qR}{a}\right)^2.\qquad(10)$$

The mean-free path λ_i may be comparable to qR ($\lambda_i \sim 10^3$ cm for typical edge parameters $T_i = 50$ eV, $n = 2\times10^{12}$) and scales as T_i^2/n. Therefore this condition is easily satisfied. On the other hand, even if this inequality is not satisfied, the decay rate from Eq. (7) is $\gamma = D_m c_s/a^2$, which is comparable to $\omega_i \sim k_\parallel c_s$, by Eq. (8). Therefore, even in the collisionless regime, Landau damping and damping by chaotic mixing are comparable. Note further that even if Landau damping dominates, it contributes to radial transport of density only because of the presence of chaotic field lines.

The initial density perturbation $\tilde{n}$ is accompanied by an initial electrostatic potential perturbation $\tilde{\phi} = T_e\tilde{n}/en$, and therefore with a radial electric field E_r. The diffusion process of Eq. (7) therefore also represents the effect of shorting out this radial electric field by the presence of stochastic field lines. The primary shorting is due to electrons and occurs on an ω_p^{-1} time scale. On longer time scales, electrons become adiabatic ($e\tilde{\phi}/T_e = \tilde{n}/n$) and the remaining potential is shorted out by the sound wave propagation. (If the density $\tilde{n}$ is specified by a particle flux from the boundary of the stochastic region, rather than as an initial perturbation, this effect represents only a tendency to short out E_r and flatten the density.) Furthermore, since $\tilde{\phi}$ is associated with an initial rotation [Eq. (3)] the diffusion coefficient (7) represents a rotation damping effect. (Here it is important that the magnetic perturbation $\delta\mathbf{B}$ be due to external coils or due to modes locked to external perturbations: a propagating magnetic perturbation would provide damping of rotation relative to a rotating frame.) This effect is of potential importance because it can balance the rotation generation mechanisms of asymmetric transport[13,14] or Reynolds stress,[15-17] which have been invoked to explain the spin-up of H-mode discharges. The present rotation damping mechanism differs from the other plausible rotation damping effect, namely magnetic pumping, in the following ways: our mechanism shorts out the electrostatic potential $\tilde{\phi}$ and therefore damps all the perpendicular motion $\tilde{\mathbf{v}}_{\perp i}$ of Eq. (3). That is, this rotation damping mechanism, which does not depend upon inhomogeneities in B, operates equally on poloidal and toroidal flows. The parallel velocity $\tilde{v}_\parallel$ propagates by means of the sound waves and eventually is redistributed. That is, $\tilde{v}_\parallel$ does not decay, in agreement with conservation of cross helicity $\int \mathbf{v}\cdot\mathbf{B}\, d^3x$ in an isothermal plasma.[18] On the other hand, magnetic pumping, which is a toroidal effect, damps out only poloidal rotation[19] and leaves toroidal rotation. From Eq. (7), we estimate the rotation damping rate in our model to be

$$\gamma_s \sim D_m c_s/a^2 \sim \left(\frac{\delta B}{B}\right)^2 \frac{qRc_s}{a^2}. \qquad (11)$$

The magnetic pumping rate is of order[19]

$$\gamma_p \sim \nu_{ii}, \quad \text{for } \nu_{ii} < v_i/qR \quad (\lambda_i > qR)$$
$$\sim v_i^2/q^2R^2\nu_{ii}, \quad \text{for } \nu_{ii} > v_i/qR. \qquad (12)$$

For the low collisionality case, the condition $\gamma_p > \gamma_s$ is exactly the condition that Landau damping be negligible. However, for the high collisionality case, the condition $\gamma_s > \gamma_p$ can be satisfied, with Eq. (9) easily satisfied (see Fig. 1). Because, as we have noted, λ_i/qR has a strong scaling T_i^2/n, and because γ_p peaks at $\nu_{ii} \sim v_i/qR$, we conclude that magnetic pumping will dominate only over a small region, and the mechanism of chaotic sound wave propagation will dominate over a much larger region, but with Landau damping contributing to the decay in the low collisionality regime.

The particle transport described by Eq. (7) is only part of the total particle loss. Note, in fact, that the energy transport due to stochastic field lines[1] is much larger than the particle transport given by Eq. (7), by a factor $v_e/c_s \sim \sqrt{m_i/m_e}$. However, experiments show that the energy confinement time is shorter than the particle confinement time, but by a much smaller factor.[20] This suggests that another source of particle transport, for example $\mathbf{E}\times\mathbf{B}$ advection due to electrostatic modes, typically dominates that due to chaotic sound wave propagation. This may be true for energy transport also. If this is the case, then the importance of sound propagation along chaotic field lines (in tokamaks with typical field errors, e.g., without an ergodic magnetic limiter) is the shorting of the radial electric field and damping of rotation. Any intrinsically ambipolar part of transport, such as electrostatic $\mathbf{E}\times\mathbf{B}$ advection, cannot contribute to these effects.

This theory of rotation damping and particle transport effect we have described can be tested in plasmas in which the perturbing field $\delta\mathbf{B}$ can be varied, for example in tokamaks with an ergodic magnetic limiter. Indeed, it has been observed[21] on TEXT that the radial electric field E_r is negative, except very near the edge. The area of positive E_r at the edge increases as the current in the ergodic magnetic limiter increases, destroying the magnetic surfaces near the edge. This sign of E_r is consistent with having adiabatic electrons and a negative density gradient in the stochastic region. In

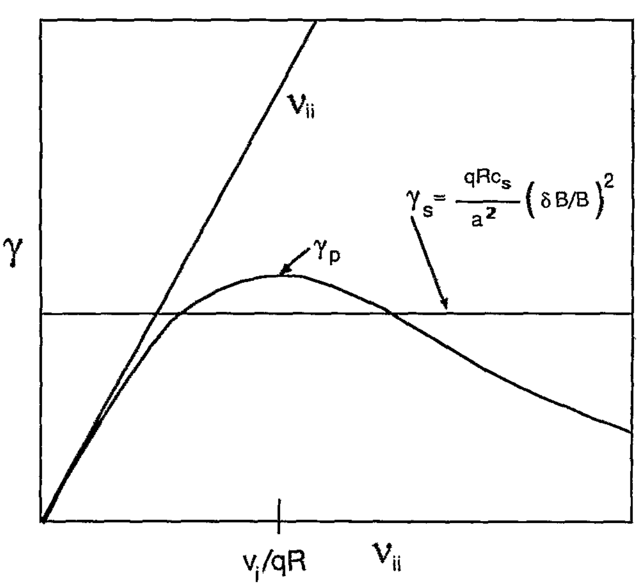

FIG. 1. A comparison of rotation damping by chaotic sound wave propagation [γ_s given by Eq. (11)] with magnetic pumping [γ_p given by Eq. (12)]. The condition for Landau damping to be negligible is $\nu_{ii} > \gamma_s$.

fact, the magnitude of the edge radial electric field has been observed to be consistent with adiabatic electrons,[21,22] in qualitative agreement with the theory.

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